

NOTATION

d, thickness of the piezoceramic cell, mm; C, velocity of sound in the ceramic, km/sec; Δt , time resolving power of the transducer, μsec ; and K_f , transmission coefficient of the solder film.

LITERATURE CITED

1. S. G. Zaitsev, "Measurement of fast varying pressures in gaseous medium," *Prib. Tekh. Eksp.*, No. 6, 97-99 (1958).
2. R. I. Soloukhin, "Piezoelectric pressure pulse transducer," *Prib. Tekh. Eksp.*, No. 3, 170-171 (1961).
3. R. I. Soloukhin, *Shock Waves and Detonations in Gases* [in Russian], Fizmatgiz, Moscow (1963).
4. Yu. E. Nesterikhin and R. I. Soloukhin, *Methods of Fast Measurements in Gas Dynamics and Plasma Physics* (Chap. 9) [in Russian], Nauka, Moscow (1967).
5. M. V. Gusev and O. I. Luneva, "Piezoelectric pressure transducer," in: *Research in Physical Gas Dynamics* [in Russian], Nauka, Moscow (1966), pp. 210-217.
6. A. P. Shatilov, "Mushroom-shaped transducer for recording the pressure in a shock tube," in: *Research in Physical Gas Dynamics* [in Russian], Nauka, Moscow (1966), pp. 218-222.
7. I. B. Sinani, "Tourmaline indicators of shock waves in fluids," *Prib. Tekh. Eksp.*, No. 4, 85-89 (1957).
8. I. S. Glushkov, N. N. Turkova, and L. I. Arutyunova, "Piezoelectric pressure transducer," *Prib. Tekh. Eksp.*, No. 5, 253-254 (1979).
9. L. G. Gvozdeva and Yu. V. Zhilin, "Piezoelectric transducer for measurement of pulsating pressures," *Prib. Tekh. Eksp.*, No. 5, 249-250 (1978).
10. K. W. Ragland and R. E. Cullen, "Piezoelectric pressure transducer with acoustic absorbing rod," *Rev. Sci. Instrum.*, **38**, No. 6, 740-742 (1967).
11. L. Bergman, *Ultrasound and Its Engineering Applications* [Russian translation], IL, Moscow (1956).

ANALYTICAL METHOD OF CALCULATING THERMAL PROCESSES AND THEIR EFFECT ON EMISSION IN SOLID-STATE LASER WITH NATURAL COOLING

G. N. Dul'nev, B. A. Ermakov,
and S. I. Khankov

UDC 621.373.8.536.242

Analytical expressions are derived for calculating the change in energy characteristics of a solid-state laser with natural cooling due to heating and thermal deformation of the active medium.

The intense heat generation in components of a solid-state laser and the significant effect of thermal processes on the emission [1, 2] call for their special analysis. Most studies made till recently dealt only with thermal processes in lasers with forced cooling. The interaction of emission processes with thermal processes is most complex in lasers with natural cooling [4, 5], but this case has not been studied sufficiently. Meanwhile, thermal processes are most pronounced in such lasers and have here a definite effect on the emission.

Operation of a laser with natural cooling is characterized by a relatively low intensity of heat transfer and, as a consequence, a long transient period. This causes the emission characteristics of the laser to change continuously from pulse to pulse, until either a thermal steady state or cutoff of emission is reached. The more important laser characteristics in this case are the limiting pulse repetition rate f_l at which the laser can operate in the steady state without emission cutoff and the limiting operation time before emission cutoff τ_l , the latter depending on the repetition rate. These parameters must be estimated for predicting the

Translated from *Inzhenerno-Fizicheskii Zhurnal*, Vol. 42, No. 2, pp. 307-313, February, 1982. Original article submitted January 20, 1981.

laser performance characteristics and this calls for analytical methods of calculation. Here will be established analytical relations for the emission energy of a laser operating under transient thermal conditions whereupon simplified expressions will be derived for the limiting repetition rate and operation time. Most thoroughly, moreover, will be treated the peculiarities of laser operation in the case of four-level low-threshold active media.

In order to calculate the change in laser emission energy due to thermal processes in the general case, it is necessary to determine the temperature field in the active medium and to establish the temperature dependence of the gain, then calculate the gain distribution over a section of the active medium and also determine the losses in the resonator resulting from the particular temperature distribution over a section of the active medium. For a derivation of analytical expressions, the pattern of thermal processes must be greatly simplified already in the mathematical formulation of the problem on each step of calculations.

As a basic assumption greatly simplifying the calculations, we let the temperature drops over a section of the active medium be negligible in comparison with the temperature level in the active medium, this assumption having been found valid in preliminary studies [4, 5]. With such an assumption, the problem reduces to determining the mean-volume temperature of the active medium, the temperature dependence of the gain, and the mode as well as the magnitude of thermal deformation of the active medium, with a subsequent determination of losses in the resonator. On the basis of an earlier analysis of the temperature field [5], it has been demonstrated that the latter influences the laser performance by affecting the properties of the resonator through two mutually independent modes of thermal deformation: wedgewise and spherical. The former mode occurs due to asymmetry of the heat transfer at the lateral surface of the active medium (heat coming from the hot bulb of the pump lamp) and the latter mode is due to internal sources of heat generation. In the absence of a fluid forced-cooling medium there will be found a predominant wedgewise thermal deformation and only a slight spherical deformation.

As a result, the well-known expression for the emission energy of a solid-state pulse laser [1] can be modified to include the effect of thermal processes on the emission

$$E = \frac{h\nu}{G} VK_r \frac{K_0[\vartheta_2(P)] - K_{t0} - \alpha_T[\beta(P)]}{K_{t0} + \alpha_T[\beta(P)]}, \quad (1)$$

where $K_{t0} = \alpha_0 + K_r$; $K_r = \frac{1}{2L} \ln \frac{1}{r}$.

Relation (1) indicates that the problem of calculating the laser emission energy reduces, in the simplified formulation, to determining the form of functions $\vartheta_2(P)$ and $\beta(P)$ as well as $K_0(\vartheta_2)$ and $\alpha_T(\beta)$.

1. The relations for the mean-volume temperatures of the optical components, viz., the pump lamp (ϑ_1), the active medium (ϑ_2), and the luminaire (ϑ_3) have been derived from the solution to a system of three differential equations describing the heat transfer from a heat source to three bodies [6]

$$\begin{aligned} \vartheta_1 &= \left(\frac{P_1}{\sigma_{13}} + \vartheta_3 \right) [1 - \exp(-m_1\tau)], \\ \vartheta_2 &= \frac{1}{1 + s_2} \left(\frac{P_2}{\sigma_{23}} + s_2\vartheta_1 + \vartheta_3 \right) \{1 - \exp[-(1 + s_2)m_2\tau]\}, \\ \vartheta_3 &= \frac{P_3 + (P_1 + P_2)[1 - \exp(-m_3\tau)]}{\sigma_{3c}l} [1 - \exp(-nm_3\tau)], \end{aligned} \quad (2)$$

where

$$\begin{aligned} m_1 &= \frac{\sigma_{13}}{C_1}; \quad m_2 = \frac{\sigma_{23}}{C_2}; \quad m_3 = \frac{\sigma_{13} + \sigma_{23}}{C_1 + C_2}; \quad m_3 = \frac{\sigma_{3c}}{C_3}; \\ s_2 &= \frac{\sigma_{12}}{\sigma_{23}}; \quad n = 1 + \kappa_2 \exp(-m_3\tau); \quad \kappa_2 = \frac{\sigma_{13} + \sigma_{23}}{\sigma_{3c}}. \end{aligned}$$

An analytical expression for the temperature ϑ_2 of the active medium can be obtained by insertion of the values of ϑ_1 and ϑ_3 into the expression for ϑ_2 , viz.,

$$\vartheta_2 = \frac{1}{1 + s_2} \left\{ \frac{P_2}{\sigma_{23}} + s_2 \frac{P_1}{\sigma_{13}} [1 - \exp(-m_1\tau)] + [1 + s_2(1 - \exp(-m_1\tau))] \times \right.$$

$$\times \frac{P_3 + (P_1 + P_2)[1 - \exp(-m_2\tau)]}{\sigma_{3e}n} [1 - \exp(-nm_3\tau)] \{1 - \exp[-(1+s_2)m_2\tau]\}. \quad (3)$$

2. The magnitude of wedgewise thermal deformation (equivalent optical wedge β in the resonator) can be calculated according to the expression [7]

$$\beta = \frac{W}{\varepsilon} \Delta\vartheta, \quad \Delta\vartheta = \frac{s_0}{1+s_0+s_2} \frac{P_1}{\sigma_{13}} \{[1 - \exp(-m_1\tau)] + \Omega_1[\exp(-m_1\tau) - \exp(-\tilde{m}_2\tau)] + \Omega_2[1 - \exp(-\tilde{m}_2\tau)]\}, \quad (4)$$

where

$$\Omega_1 = \frac{3(1-s_2)(s_0+2s_2)}{(s_0+3s_2)(s_0+4)+s_0s_2} (1+\tilde{b}); \quad \tilde{b} = 1 \left/ \left(\frac{m_1}{\tilde{m}_2} - 1 \right) \right.; \quad s_0 = \frac{\sigma_{12}}{\lambda L};$$

$$\tilde{m}_2 = \frac{12(1+s_0+s_2)}{(s_0+3s_2)(s_0+4)+s_0s_2} s_2 m_2; \quad \Omega_2 = \frac{1-s_2}{2s_2} \frac{\sigma_{13}}{\sigma_{12}} \frac{P_2}{P_1}; \quad \varepsilon = \frac{2R}{L}.$$

3. The temperature dependence of the gain in a four-level active medium can be obtained from the solution to the system of steady-state kinetic equations describing the distribution of excited electrons over energy levels [1, 8], viz.,

$$K_0 = aW_P - \alpha_T, \quad (5)$$

where

$$a = a_0/(1 + \gamma_1 b_{34}); \quad a_0 = \eta_1 \tau_L G N_0 m; \quad m = U_P B_P / W_P;$$

$$\gamma_1 = \eta A_{14} / A_{32}; \quad \alpha_T = G N_0 b_{12}; \quad b_{ij} = d_{ij} / d_{j_i} = \exp(-\Delta E_{ij} / kT).$$

The temperature dependence of the gain $K_0(\vartheta_2)$ in media with a uniformly widened luminescence line is not only a consequence of the fact that the Boltzmann coefficients b_{34} and b_{12} increase with increasing temperature but also a result of the thermal widening of the luminescence line and the attendant decrease of the cross section C for emission. The mechanism by which the luminescence line of crystalline media is widened makes the half-width $\Delta\nu$ of this line linearly dependent on the temperature [8], which permits use to represent the $\Delta\nu(\vartheta_2)$ relation in the form $\Delta\nu = \Delta\nu_0(1 + \xi\vartheta_2)$, with $\xi = (1/\Delta\nu_0)(\partial\Delta\nu/\partial\vartheta)$. Considering that G is inversely proportional to $\Delta\nu$ [8], one can easily obtain an expression for $G(\vartheta_2)$ in the form $G = G_0/(1 + \xi\vartheta_2)$, with $\Delta\nu_0$ and G_0 denoting the values of $\Delta\nu$ and G at $\vartheta_2 = 0$. In view of this, the relation $K_0(\vartheta_2)$ becomes

$$K_0 = \frac{1}{1 + \xi\vartheta_2} \left(\frac{a_0}{1 + \gamma_1 b_{34}} W_P - G_0 N_0 b_{12} \right). \quad (6)$$

This expression can be used, specifically, for estimating the maximum emission temperature. Inserting expression (6) into the threshold condition $K_0 - K_t = 0$, where $K_t = K_{t0} + \alpha_T$, we obtain

$$a_0 W_P - (1 + \gamma_1 b_{34})[(K_{t0} + \alpha_T)(1 + \xi\vartheta_2) + G_0 N_0 b_{12}] = 0. \quad (7)$$

The threshold condition (7) yields the temperature ϑ_0 of emission cutoff or the magnitude of wedgewise thermal deformation β_0 at which cutoff occurs in some special cases.

1) $K_t = K_{t0} + \alpha_T \ll G_0 N_0 b_{12}$, and $\xi = 0$. Such conditions can be realized in media with an increased concentration N_0 or a nonuniformly widened luminescence line $\xi = 0$, also at low resonator losses α_T . Emission cutoff is determined by the temperature of the active medium

$$\vartheta_0 = T_0 \frac{A_1 - \frac{\Delta E_{12}}{kT_0}}{\frac{b}{k} - A_1}, \quad A_1 = \ln \frac{G_0 N_0}{a_0 W_P - K_P}; \quad (8)$$

2) $K_t \gg G_0 N_0 b_{12}$. When $K_{t0} \gg \alpha_T \left(\frac{\partial \alpha_T}{\partial \beta} \beta \ll K_{t0} \right)$, then with $\gamma_1 b_{34} \gg \xi\vartheta_2$ ($\xi = 0$) we have

$$\vartheta_0 = T_0 \frac{\frac{\Delta E_{34}}{kT_0} - A_2}{A_2}, \quad A_2 = \ln \frac{\gamma_1}{\frac{a_0 W_P}{K_{t0}} - 1}, \quad (9)$$

and with $\gamma_1 b_{34} \ll \xi \vartheta_2$ (usually in media with a uniformly widened luminescence line and where emission cutoff occurs at a relatively low temperature) we have

$$\vartheta_0 = \frac{1}{\xi} \left(\frac{a_0 W_P}{K_{t0}} - 1 \right); \quad (10)$$

3) When $K_{t0} \lesssim \alpha_T \left(-\frac{\partial \alpha_T}{\partial \beta} \beta \gtrsim K_{t0} \right)$, $\xi = 0$, and $\gamma_1 b_{34} \ll \frac{\alpha_T}{K_{t0}}$, then emission cutoff occurs due to a wedge-wise thermal deformation of the magnitude

$$\beta_0 = \frac{a_0 W_P - K_{t0}}{\frac{\partial \alpha_T}{\partial \beta}}. \quad (11)$$

4. The gain in a three-level medium, determined from the solution to the system of kinetic equations, is

$$K_0 = \frac{1}{1 + \xi \vartheta_2} \frac{a W_P - G_0 N_0}{1 + \frac{a}{G_0 N_0} (1 + b_{23}) W_P}, \quad (12)$$

where

$$a = \frac{a_0}{1 + \gamma_2 b_{23}}; \quad a_0 = \eta_1 \tau_L G_0 N_0 m; \quad \gamma_2 = \eta \frac{A_{31}}{A_{21}}.$$

The threshold condition for a three-level medium

$$a_0 W_P - (1 + \gamma_2 b_{23}) G_0 N_0 - (1 + \xi \vartheta_2) \left[(1 + \gamma_2 b_{23}) + \frac{a_0}{G_0 N_0} (1 + b_{23}) W_P \right] (K_{t0} + \alpha_T) = 0$$

yields expressions for the temperature of emission cutoff:

1) in media with a uniformly widened line $\xi = 0$ and with $\alpha_T \ll K_{t0}$

$$\vartheta_0 = T_0 \frac{\frac{\Delta E_{23}}{k T_0} - A}{A}, \quad A = \ln \frac{\gamma_2 + \frac{K_{t0}}{G_0 N_0 + K_{t0}} \frac{a_0}{G_0 N_0} W_P}{\frac{a_0}{G_0 N_0} \frac{G_0 N_0 - K_{t0}}{G_0 N_0 + K_{t0}} W_P - 1}; \quad (13)$$

2) when $\gamma_2 b_{23} \ll \xi \vartheta_2$ and $\alpha_T \ll K_{t0}$

$$\vartheta_0 = \frac{1}{\xi} \left[\frac{G_0 N_0}{K_{t0}} \frac{\frac{a_0}{G_0 N_0} W_P - 1}{\frac{a_0}{G_0 N_0} W_P + 1} - 1 \right]; \quad (14)$$

3) when $K_{t0} \ll \alpha_T$, $\xi \vartheta_2 \ll 1$, and $\gamma_2 b_{23} \ll 1$, however, then emission cutoff is determined by the wedge-wise thermal deformation

$$\beta_0 = \frac{K_{t0}}{\frac{\partial \alpha_T}{\partial \beta}} \left[\frac{G_0 N_0}{K_{t0}} \frac{\frac{a_0}{G_0 N_0} W_P - 1}{\frac{a_0}{G_0 N_0} W_P + 1} - 1 \right]. \quad (15)$$

5. The resonator losses produced by misalignment due to wedge-wise thermal deformation can be either calculated or determined experimentally [9, 10]. According to experimental data pertaining to a resonator formed by plane-parallel mirrors, the $\alpha_T(\beta)$ relation is a direct proportion over a wide range of β (up to 5') so that one can let $\partial \alpha_T / \partial \beta = \text{const}$ for practical calculations.

6. On the basis of the obtained results, the expression for the laser emission energy (1) at every instant of time from the beginning of periodic operation without forced cooling will be in the case of a four-level active medium

$$E = \frac{h\nu}{G_0} V k_r \left\{ \frac{a_0}{1 + \gamma_1 \exp \left[-\frac{\Delta E_{34}}{k(T_0 + \vartheta_2(W_P, f, \tau))} \right]} W_P - \right.$$

$$\begin{aligned}
& -G_0 N_0 \exp \left[-\frac{\Delta E_{12} + b\vartheta_2(W_P, f, \tau)}{k(T_0 + \vartheta_2(W_P, f, \tau))} \right] - [1 + \xi\vartheta_2(W_P, f, \tau)] \times \\
& \times \left[K_{t_0} + \frac{\partial \alpha_T}{\partial \beta} \beta(W_P, f, \tau) \right] \left[K_{t_0} + \frac{\partial \alpha_T}{\partial \beta} \beta(W_P, f, \tau) \right]^{-1}, \tag{16}
\end{aligned}$$

with $\vartheta_2(W_P, f, \tau)$, $\beta(W_P, f, \tau)$ determined from expressions (3)-(4) and $W_P f = P$.

One can analyze the dependence of the laser performance during the transient thermal state on various parameters with the aid of relations (16), (3), and (4), but for practical estimates in many cases it is sufficient to determine the limiting repetition rate f_L and the limiting operation time τ_L . Using the expressions for ϑ_0 or β_0 , one can estimate the limiting pulse repetition rate f_L beyond which emission cutoff will occur or the steady thermal state will be reached as well as the limiting operation time τ_L till cutoff, both functions of f and W_P , in the case of short-time duty (high frequency f).

When $K_{t_0} \gg \alpha_T$, then

$$f_L = \frac{\vartheta_0}{W_P} \sigma_0, \quad \sigma_0 = \left(\frac{\kappa_1 + \kappa_2 + \kappa_3}{\sigma_{3c}} + \frac{\kappa_2}{\sigma_{12} + \sigma_{23}} + \frac{\sigma_{12}}{\sigma_{13}} \frac{\kappa_1}{\sigma_{12} + \sigma_{13}} \right)^{-1}, \tag{17}$$

$$\tau_L|_{f \gg f_L} = \frac{f_L}{f} \frac{1}{\kappa_2 m_2} \frac{\sigma_{23}}{\sigma_0} \Omega = \frac{C_2}{\kappa_2 W_P f} \vartheta_0 \Omega, \tag{18}$$

where $\Omega = 1$ yields an estimate of τ_L at the maximum and $\Omega = 1 / \left(1 + \frac{\kappa_1}{\kappa_2} \frac{\sigma_{12}}{\sigma_{13}} \right)$ yields an estimate of τ_L at the minimum.

When $K_{t_0} \lesssim \alpha_T$, $\xi = 0$, and $K_{t_0} \gg G_0 N_0 b_{12}$, then

$$f_L = \frac{\varepsilon \sigma_{13} (1 + s_0 + s_2) \left(a_0 - \frac{K_{t_0}}{W_P} \right)}{W \frac{\partial \alpha_T}{\partial \beta} s_0 \kappa_1 (1 + \Omega_2)}, \tag{19}$$

$$\tau_L|_{f \gg f_L} = \frac{f_L}{f} [m_1 (1 - \Omega_1) + \tilde{m}_2 (\Omega_1 + \Omega_2)]^{-1}. \tag{20}$$

7. Expressions (17) and (19) for f_L have been derived from the condition of a steady thermal state ($\tau \rightarrow \infty$), while expressions (18) and (20) for τ_L have been derived for the case of emission cutoff still during the initial stage of heating where the $\vartheta_2(\tau)$ function is nearly linear. In a strict sense, the conditions assumed for deriving the expressions (8)-(11) and (13)-(15) for ϑ_0 and β_0 rarely prevail in practice but are most often approached with some approximation. Each of expressions (8)-(11), when inserted correspondingly into expressions (17), (18), and also (19), (20) represents a dependence on one of the main factors influencing emission. This makes it possible to estimate, specifically, the most influential factor as well as the temperature ϑ_0 of emission cutoff and the limiting magnitude β_0 of the thermal deformation wedge. When proceeding to calculate f_L and τ_L , one must take into account the temperature dependence of σ_{ij} and W , their temperature dependence determining the interrelation between f_L and τ_L .

The expressions obtained here can be used not only for a qualitative analysis of the dependence of the laser emission characteristics on the thermal laser parameters but also for quantitative calculations. For instance, the limiting pulse repetition rate for two active media (glass and garnet activated with neodymium) under the condition that the energy of a pulse in the steady state be half its initial energy is found to be 0.3 Hz ($N_0 = 2 \cdot 10^{20} \text{ cm}^{-3}$, $G_0 = 3.6 \cdot 10^{-20} \text{ cm}^2$, $\tau_L = 280 \text{ } \mu\text{sec}$, $a_0 = 0.45 \cdot 10^{-2} \text{ J}^{-1} \cdot \text{cm}^{-1}$, $m = 2.2 \text{ J}^{-1} \cdot \text{sec}^{-1}$, $W/\lambda = 6 \cdot 10^{-6} \text{ W}^{-1} \cdot \text{m}$, $\xi = 0$) and 1.5 Hz ($N_0 = 1.3 \cdot 10^{20} \text{ cm}^{-3}$, $G_0 = 77 \cdot 10^{-20} \text{ cm}^2$, $\tau_L = 250 \text{ } \mu\text{sec}$, $a_0 = 3.8 \cdot 10^{-2} \text{ J}^{-1} \cdot \text{cm}^{-1}$, $m = 1.5 \text{ J}^{-1} \cdot \text{sec}^{-1}$, $W/\lambda = 10^{-6} \text{ W}^{-1} \cdot \text{m}$, $\xi = 0.01 \text{ K}^{-1}$). The experimentally determined values were 0.5 and 2 Hz, respectively. The other laser parameters used for experimental determination and for calculation were: $2R = 5 \text{ mm}$, $H = 9 \text{ mm}$, $L = 50 \text{ mm}$, $\kappa_1 = 0.5$, $\kappa_2 = 0.06$, $\kappa_3 = 0.25$ [4-7], $W_P = 2K_{t_0}/a_0$, $\sigma_{12} = 0.027 \text{ W} \cdot \text{K}^{-1}$, $\sigma_{13} = 0.07 \text{ W} \cdot \text{K}^{-1}$, $\sigma_{23} = 0.024 \text{ W} \cdot \text{K}^{-1}$, $\sigma_{3c} = 0.1 \text{ W} \cdot \text{K}^{-1}$, $\gamma_1 = 2$, and $\partial \alpha_T / \partial \beta = 1.5 \cdot 10^{-4} \text{ cm}^{-1} / \text{ang. sec}$.

NOTATION

E , laser emission energy; h , Planck constant; ν , frequency of emitted radiation; G , cross section for a radiative transition; V , volume of the active medium; r , reflection coefficient of the resonator exit mirror; K_0 , gain in the active medium; α_0 , initial loss in the resonator; α_T , resonator loss due to wedgewise thermal de-

formation; β (rad), magnitude of wedgewise thermal deformation of the active medium; $\delta_i = t_i - t_a$; t_i , temperature of the i -th component; t_a , ambient temperature; C_i , thermal capacity of the i -th component; P_i , power of heat generation in the i -th component; $\kappa_i = P_i/P$; P , average electric power of the pump; σ_{ij} , thermal conductance between components i and j ; $i, j = 1, 2, 3$ (1, pump lamp; 2, active medium; 3, luminaire); τ , time; H , distance from the axis of the pump lamp to the axis of the active medium; $2R$, diameter of the active medium; L , length of the active medium; λ , thermal conductivity of the material of the active medium; W , thermo-optical constant; U_p , spectral density of pumping radiation; B_p , corresponding Einstein coefficient; A_{ij} , probability of radiative transition from level i to level j ; d_{ij} , probability of nonradiative transition from level i to level j ; $\eta = \eta_1 \eta_2$; η_1 , quantum yield from pumping bands to metastable level; η_2 , quantum yield from the metastable level; N_0 , activator concentration; b_{ij} , Boltzmann coefficient for level j relative to level i [8]; W_p , pumping energy; τ_L , time constant of luminescence extinction; ΔE_{ij} , energy gap between levels i and j ; k , Boltzmann constant; b , an empirical coefficient accounting for the statistical sum of Stark components [8]; $T_0 = 300^\circ\text{K}$; and f , radiation pulse repetition rate.

LITERATURE CITED

1. A. A. Mak, Yu. A. Anan'ev, and B. A. Ermakov, "Solid-state lasers," *Usp. Fiz. Nauk*, 92, No. 3, 373-426 (1967).
2. B. I. Stepanov, *Methods of Laser Design* [in Russian], Nauka i Tekhnika, Minsk, Vol. 1 (1966); Vol. 2 (1968).
3. B. A. Ermakov and A. V. Lukin, "Energy parameters and thermal state of periodic-duty ruby laser," *Zh. Tekh. Fiz.*, 40, No. 7, 1418-1423 (1970).
4. I. F. Balashov, B. G. Berezin, S. F. Davydov, V. S. Kondrat'ev, and S. I. Khankov, "Temperature field in active medium of solid-state laser operating in the periodic mode without forced cooling," *Izv. Vyssh. Uchebn. Zaved., Priborostr.*, 21, No. 1, 97-101 (1978).
5. I. F. Balashov, B. G. Berezin, V. S. Kondrat'ev, and S. I. Khankov, "Thermal deformation of active medium of periodic-duty laser without forced cooling," *Izv. Vyssh. Uchebn. Zaved., Priborostr.*, 21, No. 2, 122-126 (1978).
6. G. N. Dul'nev and S. I. Khankov, "Thermal state of optical components of luminaire system of solid-state laser with natural cooling," *Inzh.-Fiz. Zh.*, 41, No. 2, 295-301 (1981).
7. G. N. Dul'nev and S. I. Khankov, "Thermal deformation of active medium of solid-state laser with natural cooling," *Inzh.-Fiz. Zh.*, 42, No. 1, 85-91 (1982).
8. A. A. Kaminskii, *Laser Crystals* [in Russian], Nauka, Moscow (1975).
9. A. P. Prishival'ko and E. K. Kopanik, "Dependence of emission parameters on adjustment of resonator mirrors," *Dokl. Akad. Nauk BSSR*, 9, No. 10, 654-658 (1965).
10. M. P. Vanyukov, V. I. Isaenko, V. P. Kalinin, and V. V. Lyubimov, "Dependence of losses in Fabry-Perot resonator on misalignment of its mirrors," *Opt. Spektrosk.*, 19, No. 2, 286-287 (1965).